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III. EFFECTS OF BUILDINGS ON THE LOCAL WIND

Wind structure in the atmospheric boundary layer

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The semi-empirical laws for the variation of mean wind speed with height and for the statistical properties of the turbulent fluctuations are briefly outlined.

Similarity considerations provide some useful ordering of the mean wind profile characteristics in relation to surface roughness and thermal stratification. Appreciable uncertainties prevail, however, especially as a consequence of the effect of thermal stratification and of variable terrain roughness.

Some generalization on similarity grounds can also be made regarding the fluctuations of horizontal wind speed as a function of roughness and stability, but there are wide variations of spectral density and scale which are not immediately explicable and which at present preclude anything more than a relatively coarse specification of the spectrum.

Features which are of special relevance to architectural aerodynamics and which are discussed briefly are:

(*a*) the difficulty of generalizing about the wind profile and turbulence *above* an urban complex;

(*b*) the requirement for estimating the magnitudes of extreme gusts as a function of mean wind speed, averaging time and height;

(*c*) the problem of generalizing about flow properties *below* roof level;

(*d*) the effect of urban airflow on the travel and dispersion of pollutants.

1. Introduction

I take my task to be that of putting before this meeting a broad picture of the general state of knowledge on the principal characteristics of the wind near the ground. Some selection of subject-matter will be essential in the time and space allowed, and naturally this selection must be made with the specific interests of the meeting in mind. However, it would be no service if I were not to make clear certain of the most general foundations for the present quantitative knowledge of wind structure, even though observationally this knowledge refers largely to sites quite different topographically from those implied by the phrase *architectural aerodynamics*.

2. The atmospheric boundary layer

By definition we are concerned with a depth of 100 m or so of the Earth's atmosphere which is usually well within the layer which we refer to variously as the *friction* layer, *Ekman* layer or *boundary* layer. Even though the depth of interest is a small fraction of the overall depth of the boundary layer there are reasons, which will become obvious, for extending an interest throughout this overall depth. The first of the foregoing terms reflects the prime importance of the aerodynamic friction arising from the motion of the air relative to the Earth's surface. The boundary layer may accordingly be regarded as the layer from which momentum is directly extracted and transferred downward to overcome this friction. Contrary to classical ideas, however, the depth of the boundary layer must be regarded as highly variable—a property which arises from the feature that the turbulent motion on which the effective transport of momentum depends is controlled both in intensity and in effective vertical range of action by the thermal stratification of the atmosphere. The consequence is that by day over land the effective top of the boundary layer is usually defined by the existence of a layer with stable density stratification (potential temperature increasing with height) beginning at a height which is usually in the range $\frac{1}{2}$ to 2 km. On a clear night with light winds, however, the stable stratification generated by ground cooling may be effective in suppressing the turbulent motion except perhaps immediately above rough ground, the effective boundary layer then being very shallow.

The forces controlling the horizontal *mean* motion of the air are the horizontal pressure gradient, the apparent deviating force which arises from reference to axes fixed on a rotating Earth and (of prime importance in the boundary layer) the horizontal shearing stresses associated with the vertical transfer of momentum to the ground beneath. The applicable Navier–Stokes equations of motion, in conventional Cartesian coordinates with x in the direction of the wind at the surface, are

$$\frac{\mathrm{d}\bar{u}}{\mathrm{d}t}-f\bar{v} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial z}(\tau_x), \tag{1}$$

$$\frac{\mathrm{d}\bar{v}}{\mathrm{d}t}+f\bar{u} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y}+\frac{1}{\rho}\frac{\partial}{\partial z}(\tau_y) \tag{2}$$

(see list of symbols).

A customary simplification of these equations is provided by restriction to homogeneous steady flow, a condition which almost never holds precisely but which may often be justifiable as an approximation, in which case the first term in each of the above equations is omitted. Two further important simplifications then follow. First, at the top of the boundary layer the last term in each equation is zero (by definition) and the two equations then define the geostrophic components of wind

$$v_g = \frac{1}{\rho f}\frac{\partial \bar{p}}{\partial x}, \tag{3}$$

$$u_g = -\frac{1}{\rho f}\frac{\partial \bar{p}}{\partial y}. \tag{4}$$

The resultant V_g, directed along the isobars, is customarily regarded as an approximation to the external or free wind unaffected by surface friction.

The second further simplification concerns small values of z, for which it may be assumed that $\partial\bar{p}/\partial x$ is independent of z and that the turning of wind direction with height is small. It then follows from integration of equation (1) that the change in resultant stress over the height range $0-z$ is

$$\frac{\tau(0)-\tau(z)}{\tau(0)} = \frac{\rho f z V_g \sin\alpha_g}{\tau(0)}, \tag{5}$$

where α_g is the total turning of wind in the boundary layer. Observations give values near 10^{-3} for $\tau_0/\rho V_g^2$ (or u_*^2/V_g^2) and 0.3 for $\sin\alpha_g$, so with $V_g = 10\,\mathrm{m\,s^{-1}}$, and $f \simeq 10^{-4}$ for middle latitudes,

$$\frac{\tau(0)-\tau(z)}{\tau(0)} \simeq 3\times10^{-3}z, \tag{6}$$

with z in metres. Thus we have a fall in stress away from the boundary which remains negligible, say less than 10 %, for heights up to about 30 m.

The preceding analysis defines for us the so-called *constant-stress layer*, a condition which has important consequences in the theoretical analysis of low-level wind structure. Following the nomenclature of Sheppard (1970) in his recent comprehensive review of the atmospheric boundary layer, we may also define an *interfacial layer* as the layer within the roughness elements in which the downward flux of momentum is handed over to pressure forces on the elements themselves. This is a layer of obviously special interest in the present context.

We now consider the detail of the wind structure in the boundary layer *above* the interfacial layer, first in respect of the mean values implicit in the foregoing equations and second in respect of the turbulent fluctuations therefrom.

3. The variation of mean velocity with height

It will be useful to begin by noting certain broad features of the wind profile, taking as an example the now well-known analysis by Lettau (1950) of a classical series of pilot balloon ascents. Figure 1 shows mean values (over a 7 h period) of the speed and direction, expressed as fractions of the geostrophic wind speed and the difference between the surface and geostrophic directions. Features which are typical of the profile over land are the rapid decrease in the speed gradient away from the ground and the relatively uniform gradient of direction over the whole depth of the boundary layer (the magnitudes of the latter justifying the approximations in the analysis defining the constant stress layer).

Historically the first analytical representation of the low-level speed profiles was in the convenient form of a simple power law $\bar{u} \propto z^q$, with the index q typically in the range 0.1 to 0.5. Although still popular in engineering applications it has long been evident from aerodynamic studies that a law of the form $\bar{u} \propto \ln z$ is a superior representation, and this form has been well established for flow in the lower atmosphere in the absence of important buoyancy forces. In general (i.e. including the buoyancy forces generated by thermal stratification of the atmosphere) the currently accepted analytical framework is provided by the similarity arguments of Monin and Obukhov.

The Monin–Obukhov similarity treatment starts with the hypothesis that in the *constant stress layer* the vertical flux/profile properties are uniquely determined by the height z and by the following four parameters

$$\rho, \quad g/T, \quad u_*, \quad H/\rho c_p, \tag{7}$$

which are implicitly independent of height. From these three scaling parameters are provided:

$$\left.\begin{array}{ll} \text{velocity} & u_*, \\ \text{temperature } (T_*) & -H/\rho c_p u_*, \\ \text{length } (L) & -\rho c_p\, T u_*^3/kgH. \end{array}\right\} \tag{8}$$

For the wind gradient dimensional considerations dictate a universal form

$$\frac{\mathrm{d}\bar{u}}{\mathrm{d}z} = \frac{u_*}{L} g\left(\frac{z}{L}\right) = \frac{u_*}{kz}\phi_{\mathrm{M}}\left(\frac{z}{L}\right) \tag{9}$$

(the numerical constant k—von Kàrmàn's constant—being defined empirically). Note that in the terms of the original argument this form refers only to the layer of constant stress and constant vertical heat flux. In integral form

$$k\bar{u}/u_* = f_{\mathrm{M}}(z/L) - f_{\mathrm{M}}(z_0/L), \tag{10}$$

where
$$f_{\mathrm{M}} = \int_{z_0}^{z} \phi(\mathrm{d} \ln (z/L)),$$

z_0 being the height at which $\bar{u}$ formally goes to zero.

In *neutral* conditions the parameters g/T and H associated with buoyancy forces must vanish from the relations, and then $\phi = 1$ and the profile takes the familiar logarithmic form

$$k\bar{u}/u_* = \ln(z/z_0) \tag{11}$$

which has been amply demonstrated in low-level profile studies over ideal sites. Values of the *roughness parameter* z_0 deduced from such studies broadly reflect the magnitude of the roughness

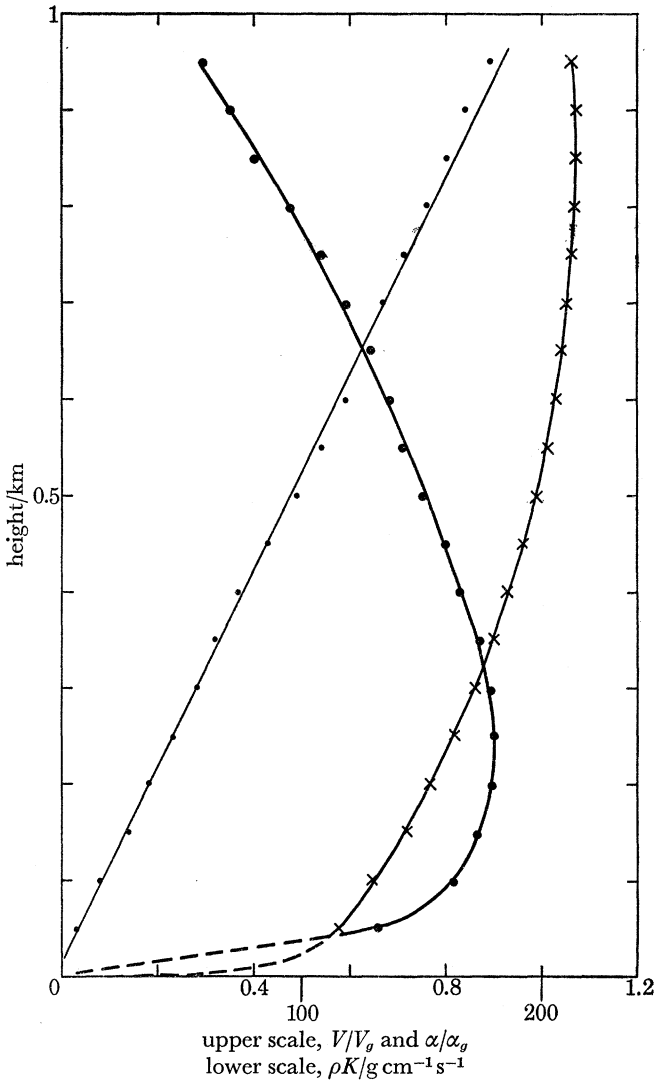

FIGURE 1. General nature of wind profile in near-neutral conditions (from re-analysis by Lettau (1950) of Leipzig pilot balloon data). ×—×, V/V_g; •, α/α_g; ●—●, ρK_m (eddy viscosity) implied. $V_g = 17.5\,\text{m}\,\text{s}^{-1}$; $\alpha_g = 26°$.

elements and range from millimetres over a very smooth area such as a cricket pitch to about 1 m from areas roughened by the presence of trees and buildings. For this range the variation of $u_*/\bar{u}(z)$ according to equation (11) is shown in figure 2*a* for two reference heights 1 and 10 m (note that the relation is meaningful only for large z/z_0). The quantity $u_*/\bar{u}(z)$ is the square root of the drag-coefficient defined by $\tau_0/\rho\bar{u}^2(z)$. Figure 2*a* indicates the increasing sensitivity of the drag coefficient to variations in roughness as the reference height is reduced. This is merely a reflexion of the relatively greater reduction of near-surface wind speed in the case of larger roughness (for a given wind speed at higher level).

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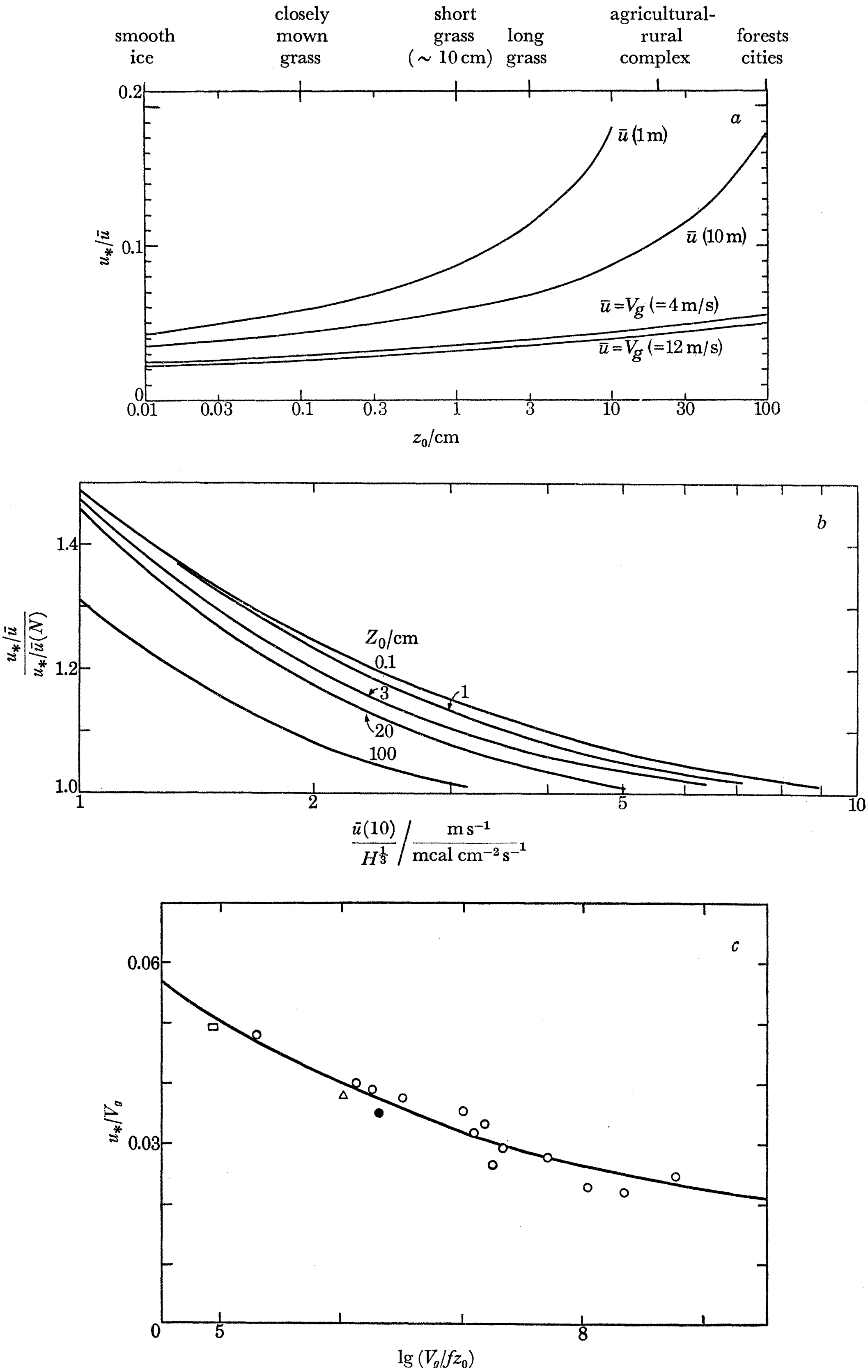

FIGURE 2. (*a*) $u_*/\bar{u}(z)$ as a function of z_0, in neutral conditions for wind speed at reference heights as indicated. (*b*) Ratio of $u_*/\bar{u}$ (at 10 m) in unstable conditions to its value in neutral conditions, as a function of z_0 (cm) and $\bar{u}(10)/H^{\frac{1}{3}}$. (*c*) u_*/V_g observed as a function of surface Rossby number Ro (after Blackadar 1962). ○, Lettau, □ △, Brookhaven, ●, Hanford.

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For stratified conditions no satisfactory *theoretical* specification of the departure of ϕ from unity has emerged. High-quality profile observations in unstable conditions indicate a systematic reduction to about 0.4 as $|z/L|$ increases to 4. Associated with this is a characteristic curvature of the $\bar{u}$, $\ln z$ shape. An illustration of the complex effect on $u_*/\bar{u}$ of heat flux H and wind speed as represented in the Monin–Obukhov length L is given in figure 2*b*, derived by numerically integrating empirical data on ϕ_M contained in publications by Swinbank & Dyer. Noting that H attains values of $420\,\mathrm{W\,m^{-2}}$ ($10^{-2}\,\mathrm{cal\,cm^{-2}\,s^{-1}}$) in middle latitudes only in the strongest insolation, it follows that important effects of instability on $u_*/\bar{u}(z)$ arise for $z = 10$ m only with quite low wind speeds at this level. This suggests that in applications such as the present, in which the greatest interest is obviously in very strong winds, the effects of unstable thermal stratification on the low-level wind profile may often be neglected.

Turning to the boundary layer in greater depth the progress in the semi-empirical development of a comprehensive analytical framework is inevitably slower. This is partly because of the loss of simplicity at heights for which the assumption of constant stress and neglect of the Coriolis term are no longer valid. It is also a consequence of the complex effect of variable terrain roughness and of the large-scale horizontal variations in temperature (the latter cause variations of horizontal pressure gradient with height). Nevertheless, a few features may be noted as having both practical value and encouraging fundamental indications.

Empirical estimates of the magnitude of u_*/V_g provide a useful starting-point. Data first assembled by Lettau (1959) and extended slightly by Blackadar (1962) are reproduced in figure 2*c*. The use of the non-dimensional parameter V_g/fz_0 (the surface Rossby number Ro) derives some support from semi-theoretical treatments and dimensional considerations, though more recent similarity arguments referred to below assign more fundamental quality to the obviously related parameter u_*/fz_0. Most of the large range of Ro arises from variations in z_0 over a range of 400 : 1. The insensitivity of u_*/V_g to z_0 is evident and is displayed directly also in figure 2*a* in curves derived from the 'eye fit' to the data in figure 2*c*, taking $f = 1.2 \times 10^4\,\mathrm{s^{-1}}$, appropriate to middle latitudes, and $V_g = 4$ and $12\,\mathrm{m\,s^{-1}}$. This has significance in also implying the variation in $\bar{u}(z)/V_g$ with z_0 for the two low levels 1 m and 10 m, and further reference is made to this point below.

The most satisfactory theoretical interpretation of the data in figure 2*c* appears to be provided, albeit in a roundabout way, by the extension of similarity arguments to a deeper section of the boundary layer. Basic analyses of this type have been given by several authors (Blackadar, Gill, Monin and Zilitinkevich) and useful summaries and appraisals of these have been provided by Smith (1968) and Sheppard (1970). Following Smith's interpretation of the arguments, with the determining parameters assumed to be z_0, u_*, V_g and f (in neutral conditions) a general similarity form may be written

$$\frac{\bar{u}}{u_*} = F\left(\frac{z}{z_0}, \frac{fz_0}{u_*}\right) = F_1\left(\frac{fz}{u_*}, \frac{fz_0}{u_*}\right). \tag{12}$$

Apparently this may be expanded in powers of the small parameter fz_0/u_* and asymptotic forms defined for two ranges of z which are then considered to overlap, namely

$$\left.\begin{aligned} &\text{small } z \quad \frac{\bar{u}}{u_*} \simeq F_2\left(\frac{z}{z_0}\right), \\ &\text{large } z \quad \frac{\bar{u}-u_g}{u_*} \simeq F_3\left(\frac{fz}{u_*}\right). \end{aligned}\right\} \tag{13}$$

Equating the two forms of $\bar{u}/u_*$ in the region of overlap, and applying similar considerations to $\bar{v}$, the following expressions emerge,

$$\left.\begin{aligned} \frac{\bar{u}}{u_*} &= \frac{1}{k}\ln\left(\frac{z}{z_0}\right), \\ \frac{u_g}{u_*} &= \frac{1}{k}\left(\ln\frac{u_*}{fz_0} - A\right), \\ \frac{v_g}{u_*} &= -\frac{B}{k}, \end{aligned}\right\} \tag{14}$$

where A and B are absolute constants which have not yet been well determined. Combination of the expressions for u_g and v_g provides u_*/V_g in terms of u_*/fz_0 and implicitly therefore in terms of V_g/fz_0.

In addition to providing a detailed basis for expecting u_*/V_g to be a function of u_*/fz_0 (or, in practice, of V_g/fz_0) the foregoing analysis also insists that the logarithmic form for $\bar{u}$ applies *only* in the overlapping region referred to above, where implicitly $z \gg z_0$. To some extent there appears to be a paradox here, since the constant-stress-layer argument implies the logarithmic form at presumably smaller values of z. On the other hand, the possibility of justifying adoption of the logarithmic profile over an extended region of the boundary layer is of considerable practical interest, especially where large roughness elements make the heights usually associated with constant stress (tens of metres) unacceptable for representative wind measurements. It is also relevant to note here the results of a current analysis by Smith (1970), which assumes the logarithmic profile of the u component of the wind to extend to the geostrophic value and also assumes $\sin\alpha$ to change linearly with height. In combination with the integral forms of the simplified equations of motion these assumptions predict a u_*/V_g relation with Ro which lies close to the observations in figure 2*c*.

A crucial remaining problem for the boundary layer as a whole is the proper incorporation of the effects of thermal stratification. Both Monin & Zilitinkevich (1967) and Blackadar (1967) claim that the expressions for u_g/u_* and v_g/u_* in equation (14) retain validity in stratified conditions, but with the parameters A and B dependent on stability. Preliminary indications of the variation of A and B are given by Monin & Zilitinkevich and, from more recent observations in Australia, by Clarke (1970). Clarke's results also provide the ratio of the quantity u_*/V to its value over the same terrain in neutral conditions, where V is the observed total wind at the level of maximum. This ratio is very scattered but there is an unmistakable sharp transition between stable and unstable conditions, and in the latter conditions its value ranges between 1 and 2 and averages about 1.6 over a wide range of instability. Extension of Smith's (1970) treatment to unstable conditions, assuming the semi-empirical profile in the constant stress layer to apply for the u component to the top of the boundary layer, gives u_*/V_g as a function of Ro and H/V_g^2. The ratio of u_*/V_g at given H/V_g^2 to its neutral value varies only slightly with Ro and for typically large values of H/V_g^2 is about 1.7, in good agreement with Clarke's average value.

The empirical background on the wind change over the whole depth of the boundary layer has also been usefully supplemented by the analysis of routine wind data by Findlater, Harrower, Howkins & Wright (1966). Their analysis provides values of the ratio of the surface wind speed (nominally at equivalent height of 10 m) to the radar wind speed ascribed to the 900 mbar (90 kN m^{-2}) level. Figure 3 shows mean values for groups defined by lapse rate, for an ocean weather station and for London (Heathrow) Airport. These data are significant in displaying rather less systematic behaviour over land than over the sea. Indeed the well-marked trend with

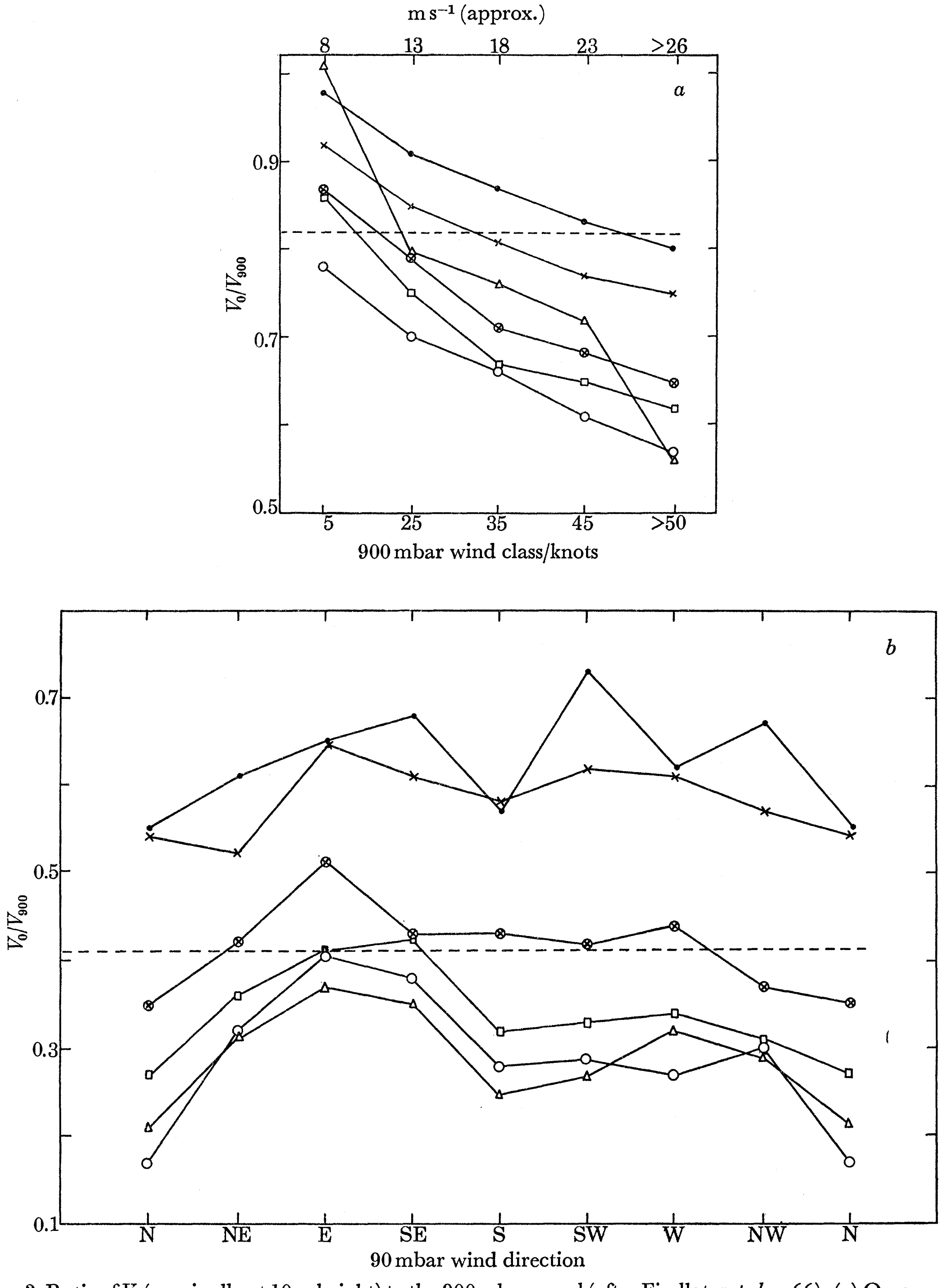

FIGURE 3. Ratio of V_0 (nominally at 10 m height) to the 900 mbar speed (after Findlater *et al.* 1966). (*a*) Ocean weather station I; (*b*) Heathrow. Broken lines are overall means. Symbols indicate class of *lapse* rate (°C per 100 m) as follows:

	● ⩾ 1	0.45 > □ ⩾ 0.2	
1	> × ⩾ 0.7	0.2 > ○ ⩾ −0.1	
0.7	> ⊗ ⩾ 0.45	−0.1 > △	

(The temperature figures having been converted and rounded from original values in °F.)

the magnitude of the 900 mbar wind over the sea is not evident over land. Instead there are marked effects of wind direction, which are considered partially to reflect upwind surface variations, though on the whole the trend with stability is in the expected direction.

Taking the data plotted as x in figure 3*b* as appropriate to neutral conditions, the V_0/V_{900} ratios lie about an average value rather less than 0.6, which on the basis of figure 2*a* implies a z_0 of about 1 cm. This is reasonably consistent with the general smoothness of an airport runway area. However, the highest values of V_0/V_{900} are near 0.64, implying a z_0 of 0.1 cm or less, which seems unduly smooth. Furthermore, comparing the most unstable class of data plotted in figure 3*b* with the neutral data it appears that the ratio $(V_0/V_{900})_U/(V_0/V_{900})_N$ is in the range 1.0 to 1.2, appreciably lower than the value of about 1.6 which would be expected from Clarke's recent analyses. A feature possibly contributing to both these discrepancies is that the stability classification used by Findlater *et al.* is in terms of a temperature difference between the 'surface' and the 900 mbar level, which means that superadiabatic conditions at low level may be included in the class accepted as neutral! This points to the difficulty of realistically generalizing about the effects of thermal stratification and emphasizes the need for detailed studies of the type described by Clarke.

4. The turbulent fluctuations of the wind

In the natural wind there appear to be fluctuations on virtually all time scales ranging from fractions of a second to many days. These variations may be expressed in the now conventional form of a spectrum, the overall form of which for the wind near the ground may be expected to be broadly as in figure 4*a*. The quantity represented is $nS_u(n)$ where

$$\int_0^\infty S_u(n)\,\mathrm{d}n = \overline{u'^2}, \tag{15}$$

u' being the fluctuation of the wind speed from its long-term average. The spectrum may be considered in three main sections—on the low-frequency side the *macrometeorological* section, associated with the large-scale patterns of airflow in the form of depressions and anticyclones—on the high-frequency side the *micrometeorological* section, associated with the small-scale fluctuations generated by the wind shears induced by surface friction—and in between a *mesometeorological* section to which presumably contribute diurnal variations of heating and variations of topography.

According to van der Hoven's (1957) well-known evaluation the macrometeorological peak occurred at about 0.01 cyc/h, with $nS(n)$ there about 5 $\mathrm{m^2\,s^{-2}}$. Minimum values in the region of 0.1 to 0.2 $\mathrm{m^2\,s^{-2}}$ are apparent at frequencies which vary mainly within the range 1 to 10 cyc/h, on the low-frequency side of the micrometeorological section. This is the area referred to as the *spectrum gap*. There seems little doubt about the existence of this feature of the spectrum but the obvious variability in its frequency location and definition tends to be overlooked. It is of course the existence of this feature which justifies the use of wind sampling times of several minutes to 1 h from which to evaluate the *mean* values considered in the earlier part of this paper. It also controls the choice of such sampling periods from which to evaluate the micrometeorological section of the spectrum, which we now consider in more detail.

An important difference between the *macro-* and *meso-* sections on the one hand, and the *micro-* section on the other is that for the former sections the turbulent energy is virtually confined to the horizontal plane (i.e. a two-dimensional field of turbulence), whereas for the latter there are important contributions in the vertical. It is indeed for the vertical component w that the frequency spectra have been most completely defined.

From observations in the first 100 m above ground the w spectrum has been found to have the

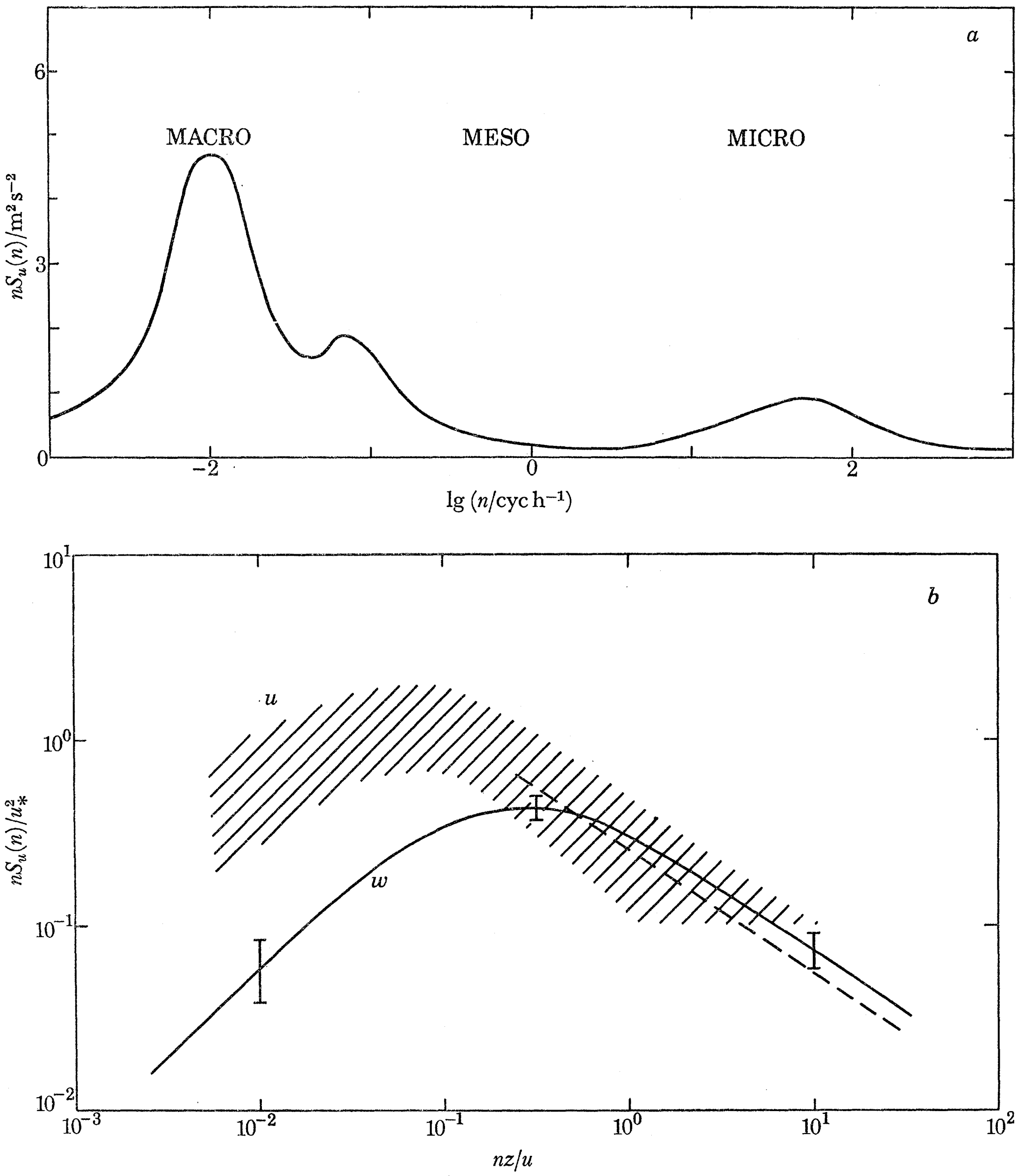

FIGURE 4. Spectra of near surface wind. (*a*) Overall form of u-component based partly on Van der Hoven (1957). (*b*) Micrometeorological spectra in first 100 m above ground. ---, equation (20).

simple form indicated in figure 4*b*. The spectral density is evidently scaled with respect to u_*^2 and the characteristic frequency with respect to $z/\bar{u}$, as predicted by application of the Monin–Obukhov similarity considerations

$$\left.\begin{aligned} nS_w(n)/\sigma_w^2 &= G(nz/\bar{u}), \\ \sigma_w/u_* &= \text{constant}\, a. \end{aligned}\right\} \qquad (16)$$

In principle functions of z/L should be included to allow for the effects of thermal stratification. It appears however (see Busch & Panofsky 1968) that the simple forms in equation (16) provide an adequate representation not only in neutral conditions but also over a practical range of unstable conditions, with the maximum value of $nS_w(n)/u_*^2$ approximately 0.4 at $nz/\bar{u}$ near 0.3. An important feature, in which differences will be found when we turn to the horizontal components, is that the low-frequency side of the w spectrum is well defined and reproducible. As regards total energy several independent estimates of σ_w/u_* in neutral and moderately unstable conditions now indicate a value near 1.25.

Although the u spectra are found to have the same general shape there are important differences. First, the simple scaling with height no longer applies and the evidence is rather confusing as regards the frequency (n_m) at which $nS(n)$ is a maximum. Until recently the bulk of evidence seemed to point to a slight systematic increase (considerably less than linear) of $\bar{u}/n_m$ with height in the first 100 m (see Berman 1965 and Busch & Panofsky 1968). However, the latest analysis by Fichtl & McVehil (1969) of data from a 150 m Nasa tower in Florida fails to show any such systematic variation. Secondly, the magnitudes of $nS_u(n)/u_*^2$ at n_m are more variable and generally between twofold and fourfold higher than those of the w component. The greater variability is in keeping with doubts regarding the applicability of the Monin–Obukhov similarity ideas to the horizontal components of turbulence. Thirdly, the energy densities on the low frequency side of n_m exhibit considerably wider variations than for the w component. This seems likely to be linked to the doubts regarding the basic applicability of the Monin–Obukhov similarity ideas and to the whole question of the origins of the energy contributions near the spectrum gap. As a reflection of the lack of order in terms of the similarity variables the u spectrum is denoted in figure 4*a* by the broad hatched area. The magnitudes of σ_u/u_* are also varied, though not to quite the same extent, a range of 2.1 to 2.9 being quoted in the literature.

As regards the variation of σ_u with height, the similarity argument implies constancy, in keeping with the effective constancy of u_* with height. For heights over which the fall of u_* may no longer be neglected Panofsky (1962) has suggested the argument that σ_u remains proportional to u_* and has applied equation (5) as a rough approximation. This means that (on a given site) the change in σ_u^2 between two levels should be proportional to the wind speed (at a given height). Panofsky quotes data in support of this from measurements made at 18 and 150 m on a tower at Cape Kennedy, Florida. Note that the application of equation (6) over such a height range implies $\sigma_u(150)/\sigma_u(18) \simeq 0.8$, whereas the results given by Harris (1968) from a single sample of turbulence measurements on a radio mast at Rugby indicate a value slightly greater than 0.9 for such a height range.

The properties of the v component also differ from those of the w component—again primarily in that there is no obvious variation of $\bar{u}/n_m$ with height, the magnitude being around 150 to 300 m, i.e. similar to that of the w component at a height of about 100 m. Also the maximum values of $nS_v(n)/u_*^2$ are variable but generally intermediate between those for the w and u components. There is a more obvious effect of thermal stratification, with the low-frequency spectral densities markedly increased in unstable conditions. On the other hand, the energy in the region of the neutral or unstable n_m appears to be virtually damped out in very stable conditions. Values of σ_v/u_* on various sites range from 1.3 to 2.6 and it has been suggested that an effect of large-scale topography (as distinct from small-scale roughness z_0) is implied.

Turning to the high-frequency region of the micrometeorological spectra, note first that it is in this region in particular that there is convincing evidence for the Taylor hypothesis relating the frequency and wavenumber representations in the form

$$nS(n) = \kappa S(\kappa) \quad \text{when} \quad n = \bar{u}\kappa. \tag{17}$$

This means that the Kolmogorov similarity treatment of the small-scale spatial structure of turbulence may in principle be applied to the corresponding range of high frequencies in the frequency spectra under consideration.

There is now rather compelling evidence that in practice there is a well-defined high-frequency section of the *u*-component spectrum fitting more or less closely to the expected variation with frequency. This follows dimensionally from the reasoning that the small-scale properties must be related uniquely to the rate of dissipation of turbulent kinetic energy ϵ. In terms of frequency

$$S_u(n) = C\bar{u}^{\frac{2}{3}}\epsilon^{\frac{2}{3}}n^{-\frac{5}{3}}, \tag{18}$$

with the universal constant C now regarded as well-determined at near 0.14 (n in Hz). Similar expressions may be written for the w and v components, but with the constant in these cases expected to be 4/3 times that for the u component.

The expected relation between the constants for the different components in the one-dimensional frequency spectra is a reflexion of the condition of local isotropy implied in the Kolmogorov theory. Physical considerations suggest that local isotropy is unlikely to exist for equivalent wavelengths ($\bar{u}/n$) similar to or larger than the height above the boundary. It is found, however, that for low heights the $-\frac{5}{3}$ region of the u component spectrum appears to hold for wavelengths up to several times the height (note however that this unexpected extension does not apply to the w component, for which a wavelength of approximately $3z$ is the peak of the $nS(n)$ spectrum and is obviously well outside the $-\frac{5}{3}$ region).

Estimates of ϵ can be obtained from a combination of the wind profile laws of the previous section and the balance equation for turbulent kinetic energy. Neglecting the diffusion of this energy the steady-state equation (for the constant stress layer) may be written

$$\epsilon = \frac{u_*^3}{kz}\left(\phi_{\mathrm{M}} - \frac{z}{L}\right). \tag{19}$$

According to Panofsky (1969*a*) an equation equivalent to (19) fits data on ϵ in unstable conditions estimated (using equation (18)) from observations on the Nasa Florida tower between 30 and 150 m.

In practice, especially at low levels, $\epsilon kz/u_*^3$ appears to change from unity surprisingly slowly as instability is increased, which may reflect a compensation from the diffusion term neglected in (19).

To sum up, for generalization on the frequency spectrum of the natural wind, the only realistic basis at present available would appear to be the combination of the similarity ideas with critical empiricism. Unfortunately, while this seems to produce a tolerably satisfactory representation of the w spectrum, the case of the u spectrum is still rather confused. Although some dependence of spectral density on u_* is evident and, furthermore, the total standard deviation σ_u at low level appears proportional to u_* within about 20 %, the details of the spectra do not otherwise closely, conform. The most that it appears justifiable to conclude at present for heights up to 100 m may be summarized as follows:

(*a*) The peak of the $nS_u(n)$ spectrum occurs at an equivalent wavelength which varies widely but is mainly in the region of 300 to 600 m.

(*b*) On *average* the spectra on the high-frequency side fit closely to a combination of equation (18) and the simple neutral form of equation (19), i.e.

$$nS_u(n)/u_*^2 = 0.26(nz/\bar{u})^{-\frac{2}{3}}, \tag{20}$$

but individual samples deviate widely from this (by factors up to 2 or more).

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(*c*) The spectral densities on the low-frequency side of the peak are especially variable.

Correlations in space vertically or laterally (acrosswind), have so far received less attention than the longitudinal (alongwind) structure implicit (with Taylor's hypothesis) in the autocorrelations at a fixed point, which provide the frequency spectra. Such data as are available indicate that in stable and neutral conditions the crosswind integral scale of the *u* component is

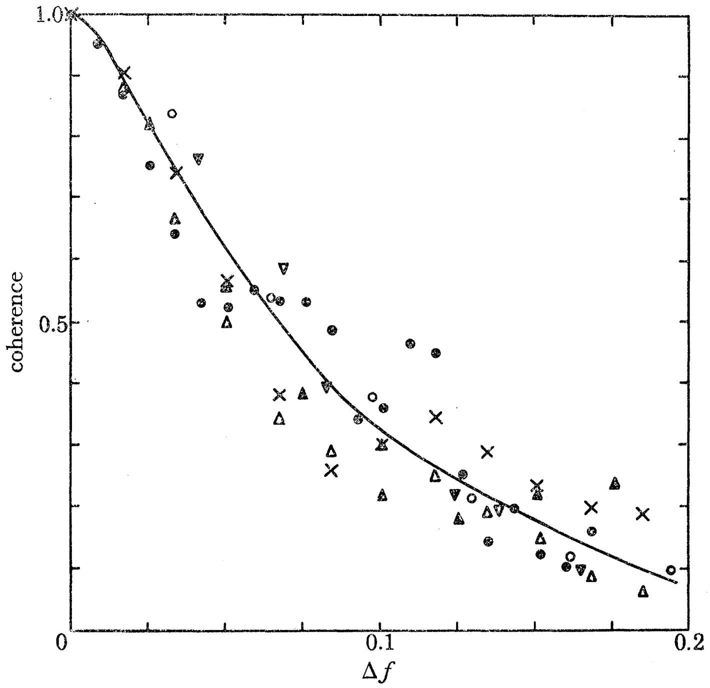

FIGURE 5. Dependence of coherence of *u*-component on separation Δ between various levels as follows:

△	22.9–53.3 m	●	38.1–53.3 m
▲	22.9–68.6 m	×	38.1–68.6 m
▼	22.9–91.4 m	○	38.1–91.4 m
▽	22.9–152.4 m		

(From data at White Sands, New Mexico, after Panofsky 1969*a* or *b*.)

less than the alongwind integral scale, by a factor of about 6 (see Panofsky 1962). In convective (light wind, unstable) conditions the difference is slight and probably negligible as a rough approximation.

Analyses of space correlation in the vertical (up to about 100 m) by Panofsky & Singer (1965) indicated (in near-neutral conditions) that correlations between various levels were a function of the *difference* in $z^{\frac{1}{3}}$, implying vertical integral scales proportional to $z^{\frac{2}{3}}$. These results also indicate phase lags between the characteristic variations at different levels, consistent with eddies having horizontal/vertical slopes between 1 and 2.

The current trend is to present the space-correlation data in terms of *coherence*, which qualitatively may be regarded as a *spectral* correlation coefficient and which is formally defined in terms of the co-spectrum, the quadrature spectrum (which are transforms of the space-correlation function) and the energy spectrum as previously discussed. Apparently the coherence for both vertical and lateral separation shows a universal relation with the ratio $\Delta f = n\Delta/\bar{u}$ where Δ is the separation and $\bar{u}$ the mean wind speed (through the layer Δ in case of vertical separation). This is in accordance with a form of geometric similarity suggested by Davenport (1961). Figure 5 shows an example taken from Panofsky (1969*a*), in which data from different pairs of levels all fall close to a single curve which approximates to simple exponential form. According to Panofsky data from various sites in neutral air, and for both vertical and lateral separation, are fitted approximately by the single relation

$$\mathrm{coh}\,(n) = \exp\left(-16n\Delta/\bar{u}\right). \tag{21}$$

5. Features of special relevance to architectural aerodynamics

Aerodynamic and wind loading problems generally assume greatest importance in extreme winds and, in many cases, over built-up areas rather than open country. It is for special consideration, therefore, how far the foregoing representations of wind structure are valid, or need to be modified, in those particular circumstances.

The focusing of interest on strong winds and large roughness elements brings simplification in one respect—namely that the effects of thermal stratification are then minimized and relations for neutral flow may in principle be expected to apply. In practice, however, there are important limitations. First, the relations apply only well above the roughness elements, which may mean that heights up to 10 to 100 m may be precluded, according to the general height reached by the buildings. Even then there is likely to be the problem of assigning a correct *zero plane* for height reference. Also, the restriction to relatively great heights may mean that the assumption of constant stress is no longer justifiable. Secondly, the relations imply a statistical uniformity in terrain roughness over an upwind fetch which, depending on the height of interest, may never be realized in typical urban areas and cities.

As regards the empirical evidence for validity over built-up areas it is true that some of the data contributing to the foregoing generalizations were in fact obtained over such terrain, but only a small fraction are in this category. For example, from the few wind profile observations over cities the estimates of the roughness parameter z_0 are in the region of 1 to 3 m (see Davenport 1961, 1968). Superficially these appear to be acceptable but their validity in relation to the limitations listed above is by no means clear.

This difficulty in generalizing about the wind profile over a built-up area is brought out by some recent observations in strong winds over Central London, obtained with instruments mounted at 195 m on the G.P.O. Tower and at 61 and 43 m on a lattice mast nearby. In an interim analysis by Shellard (1968) the results were examined in terms of a power law, which was found to be fitted only if the zero plane were taken to be somewhat above the surrounding roof tops—a modification for which it would be difficult to provide a physically rational basis. A rather larger number of records are now available and from a summary of these by Helliwell the ratios of mean wind speed for the heights 61:43 and 195:43 average 1.20 and 1.64. These fit a logarithmic profile if a zero plane displacement (d) of about 24 m is adopted (this in relation to a roof top level of 24 to 30 m in the immediate surroundings and rather greater building height at greater distance in some directions). The implied z_0 is near 0.5 m, and u_* (for $\bar{u}$ (43) 10.7 m s^{-1}) is 1.2 m s^{-1} which is reasonably consistent with the values obtained for σ_u – 2.1, 2.5 and 2.2 m s^{-1} in order of increasing height—and the range of σ_u/u_* ratios (2.1 to 2.9) previously quoted. These results are from 93 samples with $\bar{u}(43) \geqslant 8$ m s^{-1}. If progressively higher limits of $\bar{u}(43)$ are set the wind ratios progressively decrease. For example, with $\bar{u}(43) \geqslant 11$ m s^{-1}, they are 1.15 and 1.56, implying substantially smaller d and larger z_0. This is contrary to the conventional idea that for small rigid roughness elements the characteristics of the wind profile are independent of the absolute speed.

One possible explanation of the foregoing anomaly is that in stronger winds the local eddying over buildings is correspondingly more vigorous with the result that the air stream 'penetrates' more deeply between buildings, so increasing both the relative inter-building wind speed and the depth of building contributing effectively to the drag. However, there may be other factors concerned with the basic limitations already listed and the example serves primarily to illustrate the nature of the difficulties in understanding mean wind profiles and turbulence over large roughnesses. There are practical difficulties in the finding of suitable sites for the systematic investigation of such features and the variability always to be expected in atmospheric flow imposes the need to bulk large numbers of observations, a process which itself may obscure the issue and introduce unknown bias. It may be that further progress in elucidating the basic characteristics of such flows would be more economically and rapidly achieved by measurements in controlled wind tunnel flows over model surfaces.

As regards the statistical properties of the u spectrum there is no good reason at present for expecting those for an urban area in strong winds to be essentially different from the general description in the last section. Thus, if a model is required for engineering applications I am aware of no practical specification which is demonstrably superior to that set out broadly there for total σ, scale of $n\,S(n)$ peak and high-frequency spectral density.

In design considerations an obviously vital requirement is an estimate of the extreme wind speeds and fluctuations (gusts) to be expected on a particular site. This is a very complex matter in which many uncertainties call for clarification. Basically the matter may be considered in terms of an extreme mean (over say 1 h) and an extreme turbulent fluctuation therefrom over an averaging time which will depend on the nature and dimensions of the structure under load. Assuming the extreme fluctuation to be roughly proportional to σ_u for the whole spectrum of fluctuations, its behaviour in relation to averaging time is determinable from the shape of the u spectrum. Note immediately that if the high-frequency section of the spectrum conforms to equation (20) the reduction of variance in averaging over time s will be approximately proportional to $(z/\bar{u}s)^{-\frac{2}{3}}$ for an appropriate range of s. The result means that the variation of gust speed increment (over the mean speed) with averaging time should tend to follow a universal relation with $z/\bar{u}s$, though distortions from a simple relation will probably arise from variations in the whole spectrum shape and from the complex effects of wind speed noted in respect of the wind profile. A more detailed analysis would be required to test these ideas fully.

The current practice in the U.K. Meteorological Office for estimating extreme wind speeds for design purposes has been set out by Shellard (1965). The basic data are mean values over 1 h and *daily* 'extreme gusts' which have an effective averaging time of 2 to 5 s. Such data are available from a network of about 115 anemographs scattered over the U.K. As a first step the speeds are corrected to a standard height of 10 m, using empirical power laws. (Note that the exponents for gust speeds are less than those for the 1 h values, a result which can be given qualitative explanation on the following lines. As σ_u is expected to fall off slowly with height then the gust speed for infinitesimal averaging time—say $\bar{u}+m\sigma_u$ where m is a severalfold factor—must increase with height more slowly than $\bar{u}$. For finite averaging times, however, the factor m will tend to increase with height for reasons discussed in the previous paragraph. Evaluation of the net result requires a more detailed analysis, but it is noteworthy that both Shellard and Helliwell find gust speed ratios (for the pairs of heights 189/43 and 189/61 m) which are intermediate between those for $\bar{u}$ and $\bar{u}+3\sigma_u$.) The corrected data are then analysed in terms of Gumbel's theory of extreme values to define for each site the relation between the annual maxima (both 1 h and gust values) and the frequency of occurrence (or return period) thereof.

For a 50-year return period, for example, Shellard (1965) gives extreme hourly speeds ranging from below 22 m s^{-1} (50 mi/h) inland to 27–31 m s^{-1} (60–70 mi/h) on exposed coasts, and extreme gusts 40 and 45–49 m s^{-1} (90 and 100–110 mi/h) respectively. Shellard also gives empirical factors, based largely on the work of Durst, for adjusting these gust speeds to different averaging times. (It would be interesting to compare these with estimates derived from spectrum properties as discussed above.) It is particularly noteworthy that the proportionate difference between extreme *hourly* values and extreme *gusts* decreases in the order: urban areas, country sites and coastal sites—an effect of roughness which is basically represented in the drag coefficient characteristics (noting that σ_u is proportional to u_*, at least roughly).

For some purposes the region of interest will be below general roof top level—within the *interfacial* layer as defined at the beginning. The foregoing general considerations contain no quantitative guidance on this feature. The whole pattern of flow there will be a composite of the wakes, deflexions and channellings produced by the buildings and their relative dispositions. Simple effects, such as channelling when the general wind direction is near that of a straight thoroughfare lined by buildings, and stationary eddying when the wind is across such a thoroughfare, are well recognized. However, as regards the actual variations in velocity which occur in these and the many more complex situations there is no immediately available generalization. Progress to this end will require further observations of a critical nature, both on full-scale for specific sites and on model scale. From the latter in particular it may be possible to recognize reproducible features which will provide a basis for further generalization.

Finally, there is the influence of the whole complex of aerodynamic effects on the travel and dispersion of air pollutants. This aspect is dealt with in full detail in another contribution in this Symposium, but in the present context it is appropriate to note two features which arise from the larger aerodynamic drag and from the well-known urban heat-island effect. First there must be a tendency for slight upflow (and even inflow in the lightest winds) over a city area. Secondly, the increased drag and urban heating both tend to increase the turbulence in the ambient airstream and both act against the development of the ground-based nocturnal stable layer (inversion) which is characteristic of open country on clear nights, especially when the wind is light. In a recent theoretical analysis of the effect of roughness on vertical spread of airborne material released at ground level (Pasquill 1970) it was found that the roughness apparently characteristic of cities causes a vertical spread (at given distance from the source) about 3 times that over relatively smooth open country. The only experimental estimates of vertical spread over a city, those inferred from ground-level measurements of tracer concentration in St Louis, U.S.A. (McElroy & Pooler 1968) are closely consistent with the theoretical estimate.

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List of principal symbols

An overbar implies a *mean* (usually a *time* mean) value.
A subscript 0 refers to the surface or to height z_0.

c_p	specific heat of air at constant pressure
d	zero-plane correction in wind profile analysis
f	Coriolis parameter

g	acceleration due to gravity, or as a subscript denoting geostrophic value
H	vertical turbulent heat flux, positive upward
k	Von Kàrmàn's constant
L	Monin–Obukhov length
p	atmospheric pressure
Ro	surface Rossby number
$S(n)$	spectral density at frequency n
T	absolute temperature
u, v, w	velocity components along axes x, y, z
u_*	friction velocity $(=\surd(\tau_0/\rho))$
V	total wind speed in x, y plane
z_0	roughness parameter
α	angle of veer of wind with height
ρ	air density
τ	horizontal shearing stress
σ	standard deviation
ϕ_M	Monin–Obukhov stability function for momentum
ϵ	rate of dissipation of turbulent kinetic energy per unit mass

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